

On SQCD with massive and massless flavors

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ABSTRACT: We consider supersymmetric QCD in the free magnetic phase with massless and massive flavors. The theory has a supersymmetry breaking pseudo-moduli space of vacua and a runaway behavior far away from the origin. A two-loop computation reveals that the origin is destabilized and there is no meta-stable SUSY breaking solution. We also study the embedding of this model in type IIA string theory and find evidence for similar behavior. The perturbative brane dynamics involves simple interactions between branes, correctly predicting the two-loop result in the gauge theory. Our results also apply to the case when all the flavors are massive but have hierarchy among them, leading to possible instability which is manifest both in field theory and the brane description.

KEYWORDS: Supersymmetry Breaking, Brane Dynamics in Gauge Theories.

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1. Introduction

Dynamical SUSY breaking (DSB) may be an attractive explanation of the hierarchy between the electro-weak scale and the Planck scale [1]. In spite of the fact that there are models which break SUSY spontaneously at the ground state (see for example the review [2]), these models are extremely rare and they are subject to some severe constrains. Moreover, a generic (and calculable) SUSY breaking model should have a spontaneously broken R-symmetry [3], leading to a relatively light R-axion, which gets its mass only from couplings to supergravity [4] or to some higher dimensional operators. Current astrophysical observations disfavor this possibility.

These difficulties provide a good motivation to consider the possibility that SUSY is broken dynamically in a *meta-stable* vacuum (see [5] for a review of DSB both in stable and meta-stable vacua). The idea of DSB in a meta-stable vacuum got a lot of attention after it was shown by Intriligator, Seiberg and Shih (ISS) in [6] that a simple and generic class

of models, supersymmetric QCD (SQCD) in the free magnetic range, possesses local meta-stable SUSY breaking vacua (see e.g. [7, 8, 9, 10]). The authors of [6] considered SQCD with gauge group $SU(N_c)$ and N_f flavors in the range $N_c < N_f < 3N_c/2$. It was shown that if the quarks are given small and *equal* masses then the theory has a long-lived SUSY breaking vacuum. Since in this range the magnetic description of the theory is weakly coupled in the IR, the analysis at low energies was done using the Seiberg duality [11]. At tree-level massive SQCD has a moduli space of SUSY breaking vacua, but one-loop quantum effects stabilize the pseudo-moduli at the origin of field space.

In this work we study SQCD with *massless* and massive (but light) flavors. There are several reasons to consider this model. First, such a theory is an extreme case of massive SQCD with generically distributed masses. Even though the ISS model looks as a good starting point for direct gauge mediation (for early models see [12, 13, 14, 15]), it should be modified in order to produce viable phenomenology. One of the reasons for such a modification is an approximate R-symmetry at low energies which protects gauginos from getting masses. Some of the modifications included hierarchical quark masses [8] since this introduces another scale with which one can tune the lifetime independently of gaugino masses. Therefore, it is interesting to understand a limiting case of a generic distribution, taking some of the flavors to be completely massless.

Another motivation is that SQCD has a natural embedding in type IIA string theory, as the low energy theory on intersecting Neveu-Schwarz (NS) fivebranes and D-branes [16] (for a review, see [17]). The brane description of the meta-stable vacua of [6] was studied recently in [18, 19, 20, 21]. Many modifications and variations of the basic model of [6] were constructed, along with their brane descriptions (see e.g. [22, 23, 24]). It is tantalizing that in these examples one could identify identical patterns of meta-stable SUSY breaking states in the gauge theory and the classical brane system. In gauge theory, pseudo-moduli are stabilized by one-loop quantum effects [6, 7, 25], while in the classical brane dynamics regime, gravitational attraction in the NS fivebrane background stabilizes the branes in long-lived SUSY breaking meta-stable configurations [21, 24]. It is interesting to see whether this correspondence can be pushed further, and to check whether also our system has similar qualitative properties in the perturbative brane dynamics regime.

We consider $SU(N_c)$ SQCD with N_{f0} massless flavors and $N_f - N_{f0}$ massive ones in the range $0 < N_{f0} < N_c < N_f < 3N_c/2$ and study it in the dual magnetic description. In such a case the maximal possible rank of the quarks mass matrix is still larger than the rank of the dual gauge group, hence, there is no classical supersymmetric solution. Instead,

classically, these models possess a moduli space of SUSY breaking vacua. However, the pseudo-moduli associated with the massless (electric) quarks are *not* lifted by one-loop quantum effects in field theory [22] and a two-loop calculation is required to decide what is the fate of this system.

In this paper we perform the calculation of the two-loop effective potential for these pseudo-moduli. We show that at the two-loop level these directions are destabilized and, consequently, there is no SUSY breaking meta-stable vacuum near the origin. This result is also important for the case when all the flavors are massive, but there is mass hierarchy among them. In that case the two-loop contribution of heavy quarks will dominate the one-loop contribution of light quarks and the SUSY breaking solution of [6] may be destabilized.

In addition, we study the corresponding brane description and find compelling evidence that a similar instability occurs there. In particular, in the appropriate sense, the “origin” is destabilized by the brane dynamics. Note that so far in all the studied examples it was found that there is a non-trivial correspondence between the weakly coupled brane dynamics and field theory: whenever there is a meta-stable state in gauge theory one could identify a meta-stable state in the classical branes picture. Our work provides another non-trivial check of this correspondence, beyond one-loop effects in field theory (and beyond classical gravity in the brane dynamics). We emphasize that understanding the perturbative brane dynamics involves simple classical considerations, correctly predicting the result of an intricate two-loop evaluation in field theory.

This paper is organized as follows. In section 2 we study a simple Wess-Zumino (WZ) model which has a similar structure to the low energy SQCD, and then turn on the appropriate gauge interactions. In section 3 we present (after a brief review) the brane description of this gauge theory and analyze it. In section 4 we comment on some implications of our results to a theory with a general distribution of masses, especially the issue of stability of the local minimum of massive SQCD and its brane construction. Finally, we summarize in section 5. Appendix A contains a brief review of [26] and some technical details related to our calculation.

2. Field Theory Analysis

2.1 A Simplified Wess-Zumino Model

In this section we analyze a simple WZ model, which has a pseudo-moduli space of SUSY breaking vacua at the one-loop approximation, and show that it *does not* have any SUSY

	$U(1)_{\chi\rho}$	$U(1)_\sigma$	$U(1)_{\tilde{\sigma}}$	$U(1)_R$
$\Phi_{11}, \Phi_{12}, \Phi_{21}, \Phi_{22}$	0	0	0	2
X, Y	1	0	-1	2
\tilde{X}, \tilde{Y}	-1	-1	0	2
χ, ρ	-1	0	0	0
$\tilde{\chi}, \tilde{\rho}$	1	0	0	0
σ	0	1	0	0
$\tilde{\sigma}$	0	0	1	0
Z	0	-1	-1	2

Table 1: The chiral superfields and their global $U(1)$ charges.

breaking minimum near the origin of field space. Consider a model with the chiral superfields listed in Table 1, and a superpotential

$$\mathcal{W} = h q^i \Phi_i^j \tilde{q}_j - h \mu^2 (\Phi_{11} + \Phi_{22}), \quad (2.1)$$

where $i, j = 1, 2, 3$. The components of the matrix Φ and vectors q, \tilde{q} are given in terms of the fields in Table 1 by

$$\Phi = \begin{pmatrix} \Phi_{11} & \Phi_{12} & X \\ \Phi_{21} & \Phi_{22} & Y \\ \tilde{X} & \tilde{Y} & Z \end{pmatrix}, \quad q = \begin{pmatrix} \chi & \rho & \sigma \end{pmatrix}, \quad \tilde{q} = \begin{pmatrix} \tilde{\chi} \\ \tilde{\rho} \\ \tilde{\sigma} \end{pmatrix}. \quad (2.2)$$

Let the Kähler potential be canonical. The parameter h controls our loop expansion. Of course, the physical parameter corresponding to h is IR free, allowing a faithful perturbative treatment.

The model has manifest $SU(2) \times U(1)_{\chi\rho} \times U(1)_\sigma \times U(1)_{\tilde{\sigma}} \times U(1)_R$ symmetry, where the two upper components of q and \tilde{q} transform as fundamentals of the $SU(2)$ and Φ transforms in the adjoint.¹ Under the various $U(1)$ symmetries the fields transform as summarized in Table 1. Note that once μ is turned off there is an $SU(3)^2 \times U(1)_B \times U(1)_R$ symmetry. The baryon number is still present in our model as $U(1)_{\chi\rho} + U(1)_{\tilde{\sigma}} - U(1)_\sigma = U(1)_B$.

This system has no classical SUSY preserving vacuum. The F-terms of the relevant

¹By that we mean that the upper-left 2×2 submatrix of Φ sits in the adjoint, (X, Y) and (\tilde{X}, \tilde{Y}) are fundamentals of $SU(2)$ and Z is neutral.

meson components are

$$\frac{\partial \mathcal{W}}{\partial \Phi_{ij}} = h \begin{pmatrix} \tilde{\chi} \\ \tilde{\rho} \end{pmatrix} \begin{pmatrix} \chi & \rho \end{pmatrix} - h\mu^2 \mathbb{I}_{2 \times 2}, \quad i, j = 1, 2. \quad (2.3)$$

The first term is at most of rank one while the second term is of rank two. Thus, SUSY is broken by a rank condition. Nonetheless, there is a stationary point with positive energy (i.e. spontaneously broken SUSY),

$$\Phi = 0, \quad q = \begin{pmatrix} \mu & 0 & 0 \end{pmatrix}, \quad \tilde{q} = \begin{pmatrix} \mu \\ 0 \\ 0 \end{pmatrix}. \quad (2.4)$$

The global symmetry is broken as $SU(2) \times U(1)_{\chi\rho} \hookrightarrow U(1)'$. Thus, the above classical solution has three Goldstone bosons. In addition, classically, there are some dangerous pseudo-flat directions. Our purpose is to understand their quantum mechanical fate. For convenience, we take μ and h to be real and define

$$\rho_{\pm} = \frac{1}{\sqrt{2}}(\rho \pm \tilde{\rho}), \quad \chi_{\pm} = \frac{1}{\sqrt{2}}(\chi \pm \tilde{\chi}). \quad (2.5)$$

The squared mass of the fields $\sigma, \tilde{\sigma}, X, \tilde{X}, \Phi_{12}, \Phi_{21}$ is $h^2\mu^2$. Similarly, the mass of χ_+, Φ_{11} , $\Im\rho_+, \Re\rho_-$ is $2h^2\mu^2$. The three real Goldstone bosons are $\Im\chi_-, \Re\rho_+, \Im\rho_-$. All the rest are “accidental” pseudo-moduli which are not protected quantum mechanically, in general.

The results of the one-loop effective potential in this model are known from [22] and we shall review them here. All pseudo-moduli fields but Z obtain similar positive mass squared terms,

$$m_{\Re\chi_-}^2 = m_{\Phi_{22}}^2 = 2m_{Y, \tilde{Y}}^2 = h^4\mu^2 \frac{\ln 4 - 1}{8\pi^2}.$$

However, Z remains massless at one-loop. One can argue that in the one-loop effective potential there will be no Z^n terms, for any $n > 1$. The way to see it is to turn off the expectation value of the classical pseudo-moduli Y and \tilde{Y} . Doing so, particles whose mass depends on the expectation value of Z are decoupled (in the mass matrix) from the ρ sector which breaks SUSY. Thus, they sit in supersymmetric multiplets and the one-loop contribution vanishes identically.

This means that in order to understand the dynamics of this model it is necessary to compute the two-loop effective potential along the pseudo-moduli space parameterized by Z . Explicitly, we replace all the fields by their fluctuations and assume, without loss of generality, that the field Z obtains a real expectation value around which it fluctuates.

Indeed, we can use the symmetry generator $U(1)_\sigma + U(1)_{\tilde{\sigma}}$ to rotate the point where Z is real to any other complex value of Z with the same magnitude (Note that all the other expectation values vanish since they correspond to fields which are massive at tree-level or one-loop.).

The superpotential is given by

$$\begin{aligned} \mathcal{W} = & -h\mu^2\delta\Phi_{22} - h\mu^2\delta\Phi_{11} + \\ & + \frac{h}{2} \begin{pmatrix} \delta\chi_+ + \delta\chi_- + \sqrt{2}\mu \\ \delta\rho_+ + \delta\rho_- \\ \sqrt{2}\delta\sigma \end{pmatrix}^T \begin{pmatrix} \delta\Phi_{11} & \delta\Phi_{12} & \delta X \\ \delta\Phi_{21} & \delta\Phi_{22} & \delta Y \\ \delta\tilde{X} & \delta\tilde{Y} & Z + \delta Z \end{pmatrix} \begin{pmatrix} \delta\chi_+ - \delta\chi_- + \sqrt{2}\mu \\ \delta\rho_+ - \delta\rho_- \\ \sqrt{2}\delta\tilde{\sigma} \end{pmatrix}. \end{aligned} \quad (2.6)$$

The spectrum of masses is as quoted above (when expanding around $Z = 0$), except that the fields $\sigma, \tilde{\sigma}, X, \tilde{X}$ mix in a simple manner. The mass eigenstates are given by some linear combinations

$$\delta\sigma = \sin\theta \delta A + \cos\theta \delta B, \quad \delta X = \cos\theta \delta A - \sin\theta \delta B, \quad (2.7)$$

and analogous equations for the tilded fields (with the same mixing angles). Hereafter we use the notations $s_\theta \equiv \sin\theta$ and $c_\theta \equiv \cos\theta$. A and B are mass eigenstates with the following masses

$$m_{A,B}^2(Z) = h^2 \left(\mu^2 + \frac{Z^2}{2} \mp \frac{Z}{2} \sqrt{Z^2 + 4\mu^2} \right). \quad (2.8)$$

The mixing angle is

$$s_\theta^2 = \frac{h^2\mu^2 - m_A^2}{m_B^2 - m_A^2}. \quad (2.9)$$

The mass spectrum of all the particles except ρ_\pm is supersymmetric.

From now on, the two-loop evaluation is, in principle, straightforward (but in practice there are many diagrams). All the required two-loop functions and diagrams are beautifully described in [26]; some highlights are reviewed in Appendix A. We have simplified the computational task (in particular, the number of diagrams) with a few tricks which may be useful also in other models.

Consider a different theory in which we switch off the linear term for Φ_{22} in the superpotential \mathcal{W} . In other words, we consider a theory whose superpotential is $\mathcal{W}' = \mathcal{W} + h\mu^2\delta\Phi_{22}$, where \mathcal{W} is given by (2.6). In this model the moduli space parameterized by Z still exists but now it is a supersymmetric moduli space. It cannot be lifted by perturbative quantum corrections. This means that the effective two-loop potential vanishes identically as a function of Z . Thus, we can write the trivial equation,

$$V_{\mathcal{W}}^{(2)} = V_{\mathcal{W}}^{(2)} - V_{\mathcal{W}'}^{(2)}, \quad (2.10)$$

for the two-loop effective potential we are after, $V_{\mathcal{W}}^{(2)}$. Note that all of the Yukawa, cubic and quartic interactions are identical in the two models. In fact, the only difference is that the fields ρ_{\pm} of the model \mathcal{W}' are in supersymmetric multiplets with mass $h^2\mu^2$. Consequently, diagrams that do not cancel on the right hand side of (2.10) contain necessarily a ρ_{\pm} scalar. However, this is not the only simplification we can make. Since we want the diagrams to have some Z dependence, we should better have an A or B (fermion or boson) running in the loop. Otherwise, the diagram contributes only to the overall zero-point energy which we are not interested in. In this way we remain with only three different diagrams! They are depicted in Fig. 1.

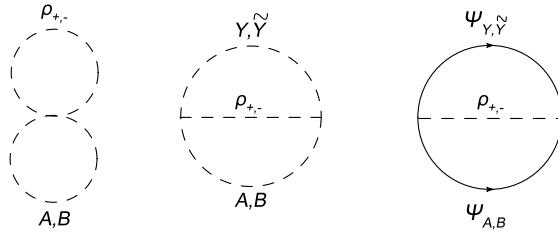


Figure 1: The only 3 two-loop diagrams contributing to the Z dependent part of the effective potential. Conforming with [26], we refer to them as SS, SSS and FFS, respectively.

Note that in general there is another possible topology for a two-loop diagram, one that includes mass flips for fermions (the third diagram depicted in Fig. 4). Even though the fermionic mass term in our theory is not diagonal this diagram is absent because there is always a ψ_Y fermion in the loop, which is massless.

At this stage, it remains to evaluate the coefficients of the diagrams in Fig. 1. Of course, as follows from (2.10), we must subtract from each of the diagrams the corresponding diagram in the theory \mathcal{W}' . In terms of the functions given in [26] and reviewed in Appendix A, we get

$$V^{(2)} = V_{SS}^{(2)} + V_{SSS}^{(2)} + V_{FFS}^{(2)}, \quad (2.11)$$

where

$$V_{SS}^{(2)} = h^2 s_{\theta}^2 (f_{SS}(2h^2\mu^2, m_A^2) - 2f_{SS}(h^2\mu^2, m_A^2)) + (A, s_{\theta}^2) \leftrightarrow (B, c_{\theta}^2), \quad (2.12)$$

$$\begin{aligned} V_{SSS}^{(2)} = h^4 (\mu c_{\theta} + Z s_{\theta})^2 & (f_{SSS}(0, 0, m_A^2) + f_{SSS}(0, 2h^2\mu^2, m_A^2) - 2f_{SSS}(0, h^2\mu^2, m_A^2)) + \\ & + (A, c_{\theta}, s_{\theta}) \leftrightarrow (B, -s_{\theta}, c_{\theta}), \end{aligned} \quad (2.13)$$

$$V_{FFS}^{(2)} = h^2 s_\theta^2 (f_{FFS}(0, m_A^2, 0) + f_{FFS}(0, m_A^2, 2h^2 \mu^2) - 2f_{FFS}(0, m_A^2, h^2 \mu^2)) + \\ + (A, s_\theta^2) \leftrightarrow (B, c_\theta^2). \quad (2.14)$$

Our results manifestly look like the difference of amplitudes in two different theories. From here on, it is straightforward to expand these functions in a Taylor series and to get that the overall contribution to the effective potential is (promoting Z to be complex again)

$$V^{(2)} = \text{const} + h^6 \mu^2 \left(-1 - \frac{\pi^2}{6} + \ln 4 \right) |Z|^2 + \mathcal{O}(|Z|^4). \quad (2.15)$$

Thus, the origin is destabilized. An examination of the effective potential as a function of Z shows that there is no minimum around the origin; the effective potential decreases monotonically.

Not surprisingly, there is no dependence on the renormalization scale, Q , in front of $|Z|^2$. It is a consequence of the following RGE argument. The effective potential satisfies an equation of the schematic form

$$\left(Q \frac{\partial}{\partial Q} + \beta_h \frac{\partial}{\partial h} - \gamma_\phi \phi \frac{\partial}{\partial \phi} \right) V = 0,$$

where β_h is the beta-function of a (physical) coupling h and γ_ϕ is the anomalous dimension of a field ϕ . Since both of these functions begin at one-loop order (or higher), and since there is no $|Z|^2$ term at one-loop or at tree-level, $Q \frac{\partial}{\partial Q}$ should annihilate the two-loop coefficient of $|Z|^2$ (or higher powers of Z), as indeed happens. Such an RGE argument is more general: loosely speaking, this means that whenever a physical effect appears for the first time in the effective potential, it must be renormalization scheme independent.

2.2 Supersymmetric QCD

Our general model is SQCD, whose UV electric description is given by the superpotential

$$\mathcal{W} = \sum_{a=1}^{N_f} m_{(a)} Q_a \tilde{Q}^a, \quad (2.16)$$

where Q_a (\tilde{Q}_a) is in the (anti-)fundamental representation of the gauge group $SU(N_c)$. We choose $N_c < N_f < 3N_c/2$, where the theory is in the free magnetic phase, and we take N_{f0} of the flavors to be massless, such that $0 < N_{f0} < N_c$. This implies that, non-perturbatively, far away from the origin, the theory has a runaway potential for the mesons associated with massless quarks (see, for instance, the reviews [27, 28]). The other $N_f - N_{f0}$ flavors are massive but much lighter than the strong coupling scale.

One can analyze this theory in the IR by using the Seiberg duality [11], which transforms the model above to an $SU(N \equiv N_f - N_c)$ gauge theory and matter content of a gauge neutral $N_f \times N_f$ meson matrix Φ_i^j and N_f flavors of (anti-)fundamental dual quarks q^i (\tilde{q}_j). The superpotential is

$$\mathcal{W} = hTr'(q^i \Phi_i^j \tilde{q}_j) - h\mu^2 Tr(\Phi_{11} + \Phi_{22}) + \text{non-perturbative}, \quad (2.17)$$

where Tr' is taken over the N color indices, Tr is over flavor indices, and we parameterize

$$\Phi = \begin{pmatrix} (\Phi_{11})_{N \times N} & \Phi_{12} & X \\ \Phi_{21} & (\Phi_{22})_{(N_c - N_{f0}) \times (N_c - N_{f0})} & Y \\ \tilde{X} & \tilde{Y} & (Z)_{N_{f0} \times N_{f0}} \end{pmatrix},$$

$$q^T = \begin{pmatrix} \frac{1}{\sqrt{2}}(\chi_+ + \chi_-)_{N \times N} \\ \frac{1}{\sqrt{2}}(\rho_+ + \rho_-)_{(N_c - N_{f0}) \times N} \\ (\sigma)_{N_{f0} \times N} \end{pmatrix},$$

$$\tilde{q} = \begin{pmatrix} \frac{1}{\sqrt{2}}(\chi_+ - \chi_-)_{N \times N} \\ \frac{1}{\sqrt{2}}(\rho_+ - \rho_-)_{(N_c - N_{f0}) \times N} \\ (\tilde{\sigma})_{N_{f0} \times N} \end{pmatrix}. \quad (2.18)$$

Note that the model in the previous subsection amounts to the case $N_{f0} = N_c - N_{f0} = N_f - N_c = 1$.

Again, rank conditions force us to expand around a SUSY breaking vacuum, as in (2.4). Indeed, considering the F-terms for Φ_i^j , the rank from the cubic superpotential coupling is at most $N = N_f - N_c$ while the rank from the linear terms in the superpotential is $N_f - N_{f0}$. As long as $N_{f0} < N_c$ we cannot balance these terms and SUSY is classically broken. Interestingly, this condition is also necessary and sufficient for runaway behavior, which is induced by non-perturbative dynamics.

So, the system settles into a SUSY breaking solution of the equations of motion,

$$q^T = \begin{pmatrix} \mu \mathbb{I}_{N \times N} \\ 0 \\ 0 \end{pmatrix}, \quad \tilde{q} = \begin{pmatrix} \mu \mathbb{I}_{N \times N} \\ 0 \\ 0 \end{pmatrix}. \quad (2.19)$$

Expanding around this solution we discover, not surprisingly, a plethora of massive and massless modes, very similar to the toy model analyzed in the previous subsection. A notable field is, of course, the Z matrix which remains massless even after a one-loop calculation for the same reasons as in our simplified model. All the other modes are either

massive at tree-level or gain some positive mass squared at one-loop. An unimportant technical difference from the toy model is that now $\Im\chi_-$, $\Re\chi_-$ are eaten by the supersymmetric Higgs mechanism.²

Thus, again, we need to understand the dynamics of the Z field near the origin. The global symmetry is $SU(N_f - N_{f0}) \times U(N_{f0})_\sigma \times U(N_{f0})_{\tilde{\sigma}} \times U(1)_{\chi\rho} \times U(1)_R$ and is spontaneously broken in the state (2.19) to $SU(N_c - N_{f0}) \times U(N_{f0})_\sigma \times U(N_{f0})_{\tilde{\sigma}} \times U(1)' \times U(1)_R$. The gauge symmetry $SU(N_f - N_c)$ is completely Higgsed. We can use the subgroup $U(N_{f0})_\sigma \times U(N_{f0})_{\tilde{\sigma}}$ to diagonalize Z and make the eigenvalues real. This simplifies the mass matrix along pseudo-moduli space considerably. It is actually just several copies of the one we considered in the previous subsection. In particular, these symmetry considerations imply that the quadratic term takes the form $V^{(2)} \sim \text{Tr}(Z^\dagger Z)$, so to determine its coefficient it is enough to turn on a single eigenvalue which is what we do in the following.

Let us first consider the non-gauge interactions. For the purpose of the two-loop computation, it is straightforward to see that this model breaks up into $N(N_c - N_{f0})$ copies of the basic interactions we considered in the simplified model. A straightforward way to see that is to reconsider any of the diagrams depicted in Fig. 1, e.g. the second diagram. There are $(N_c - N_{f0})$ possible Y mesons (since only one eigenvalue of Z is turned on) and the color of the squarks has to be matched and summed over, so we get another factor of N . Similar counting applies to the other two diagrams.

Now we have to turn on gauge interactions. The basic observation here is that the spectrum of vector multiplets is supersymmetric over the whole moduli space [6]. The reason is that gauge symmetry is broken in the sector of χ which is decoupled in the mass matrix from SUSY breaking. Thus, our criteria that there has to be a ρ_\pm scalar and either A or B particles are still applicable. In these circumstances, since gauge interactions are flavor diagonal, there are no vertices containing, for instance, A_μ, ρ, σ . Hence, there are no diagrams with particles from vector multiplets which contribute to powers of Z in the effective potential. One could worry about new interactions between quarks from D-terms,

$$V_D = \frac{g^2}{2} \sum_A \left(\text{Tr} q^\dagger T_A q - \text{Tr} \tilde{q}^\dagger T_A \tilde{q} \right)^2,$$

where the trace is over flavor indices. In the mass basis one can see that all the interactions $R_1^2 R_2^2$, where R_1 and R_2 are real scalars, cancel. So, there are no relevant contributions

²However, the trace part remains massless as long as baryon symmetry is ungauged. The real part of the trace becomes massive via one-loop effects and the imaginary part of the trace is an exact Goldstone boson.

either from gauge interactions or from D-terms (Intuitively, we do not expect non-trivial effects from D-terms in the absence of accompanying fermionic loops.).

We conclude that this gauge theory exhibits instability near the origin, with no nearby minimum, plausibly sloping to the runaway at large values of the Z meson. The effective potential in the Z direction takes the explicit form

$$V = h^6 \mu^2 \frac{N(N_c - N_{f0})}{(16\pi^2)^2} \left(-1 - \frac{\pi^2}{6} + \ln 4 \right) \text{Tr}(Z^\dagger Z) + \mathcal{O}((Z^\dagger Z)^2). \quad (2.20)$$

3. Brane Embedding

We now embed the gauge theory of subsection 2.2 on intersecting branes in the type IIA string theory. In subsection 3.1 we present the brane construction and review the mapping of its parameters to gauge theory. In subsection 3.2 we describe the perturbative brane dynamics – the *classical* forces between the branes – and its interplay with the perturbative *quantum* dynamics found in gauge theory.

3.1 Brane Configuration

To construct the brane configurations in type IIA it is convenient to decompose the $9 + 1$ dimensional spacetime as follows:

$$\mathbb{R}^{9,1} = \mathbb{R}^{3,1} \times \mathbb{C}_v \times \mathbb{R}_y \times \mathbb{R}_{x^7} \times \mathbb{C}_w. \quad (3.1)$$

The $\mathbb{R}^{3,1}$ is in the directions (x^0, x^1, x^2, x^3) , common to all the branes. The complex planes \mathbb{C}_v , \mathbb{C}_w and the real line \mathbb{R}_y correspond to

$$v = x^4 + ix^5, \quad w = x^8 + ix^9, \quad y = x^6. \quad (3.2)$$

We begin with the brane configuration of Fig. 2(a), whose low energy limit is the magnetic theory described in the previous section with $\mu = 0$ [16] (for a review, see [17]).

Fig. 2(a) presents a two dimensional slice (x, y) , where x is a certain direction in v . The line at the bottom of the figure stands for an NS5 brane, which is stretched in the direction v and located at $y = x^7 = w = 0$. We shall call it the NS brane. The bullet stands for another NS5 brane, which is stretched in the direction w and located at $v = x^7 = 0$ and $y = y_1 > 0$. We call it the NS' brane. The \times denotes a stack of N_f D6 branes, which are extended in the (x^7, w) space and located at $v = 0$ and $y = y^2$; note that $y_2 > y_1$. These are all the extended branes involved in our configurations.

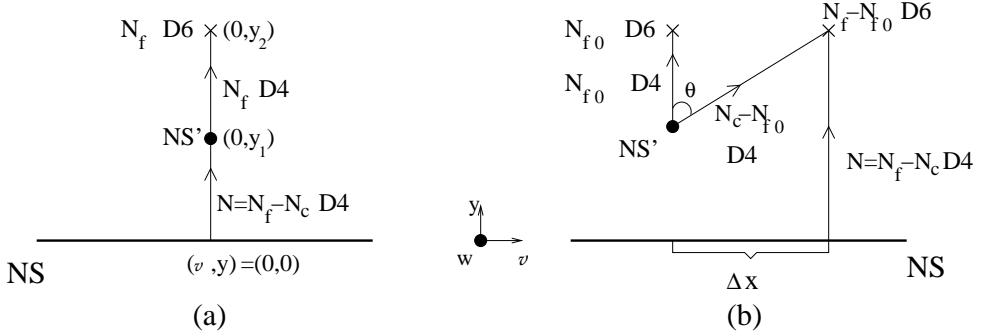


Figure 2: (a) is the brane construction of the magnetic theory with massless quarks. (b) describes its deformation by a non-zero μ parameter. In the electric language, N_{f0} flavors are still massless after this deformation.

We also have D4 branes which are stretched between extended branes. There are $N = N_f - N_c$ D4 branes stretched between the NS and NS' branes, and N_f D4 branes are stretched between the NS' and D6 branes. Arrows on the D4 branes indicate their orientation.

The low energy theory on the N D4 branes stretched between the fivebranes is 3+1 dimensional $\mathcal{N} = 1$ SYM with gauge group $U(N)$. Strings stretched between these N “color D4 branes” and the N_f “flavor D4 branes” correspond to N_f fundamental chiral superfields q^i, \tilde{q}_i . Strings whose both ends lie on the flavor D4 branes give rise to gauge singlet superfields Φ_i^j . These are coupled via the superpotential

$$W_{\text{mag}} = h q^i \Phi_i^j \tilde{q}_j. \quad (3.3)$$

This magnetic theory is the Seiberg dual of $U(N_c)$ SQCD with N_f massless flavors [11].

The mapping between the parameters of the brane construction and the gauge theory is the following. The classical $U(N)$ gauge coupling g_{mag} is given by

$$g_{\text{mag}}^2 = \frac{g_s l_s}{y_1}, \quad (3.4)$$

where g_s and l_s are the string coupling and length, respectively. The Yukawa coupling h is given by

$$h^2 = \frac{g_s l_s}{y_2 - y_1}. \quad (3.5)$$

Finally, the superpotential (3.3) has flat directions corresponding to arbitrary expectation values of Φ while setting $q = \tilde{q} = 0$. In the brane picture, giving an expectation value $\langle \Phi_i^i \rangle$ corresponds to moving the i 'th flavor D4 brane to the location w_i between the NS'

and D6 branes. A non-zero expectation value $\langle \Phi_i^i \rangle$ gives a mass $h\langle \Phi_i^i \rangle$ to the quarks q^i, \tilde{q}_i . Geometrically, this corresponds to the length of a string stretched between the i 'th flavor brane and the color branes, and hence,

$$h\langle \Phi_i^i \rangle = \frac{w_i}{2\pi l_s^2}. \quad (3.6)$$

Another deformation of this brane configuration, which is the main focus of this work, is to displace a stack of $N_f - N_{f0}$ out of the N_f D6 branes relative to the NS' brane in the v direction. The resulting configuration is shown in Fig. 2(b). The separation between the $N_f - N_{f0}$ D6 branes and the NS' brane is denoted by Δx . This brane system is the one studied in [18, 19, 20, 21]; the latter focus on the special case $N_{f0} = 0$, while here we take $0 < N_{f0} < N_c$. The configuration of Fig. 2(b) is the energetically favorable one.

After displacing the $N_f - N_{f0}$ D6 branes, only N_{f0} of the flavor D4 branes, the “massless flavor branes,” stay at their original position (with respect to NS'). On the other hand, $N \equiv N_f - N_c$ of the flavor branes connect to the N color branes and move with them to the position $v = \Delta x$, where they are stretched between the NS and the $N_f - N_{f0}$ D6 branes in the y direction. The remaining $N_c - N_{f0}$ flavor D4 branes remain stretched between the NS' and the $N_f - N_{f0}$ D6 branes, and hence are tilted in the (y, v) space.

In the low energy gauge theory, this deformation amounts to adding to the magnetic theory (3.3) a linear superpotential giving rise to the theory studied in the previous section, (2.17). The mass parameter μ in the gauge theory is related to Δx by [20]

$$\mu^2 = \frac{\Delta x}{g_s l_s^3}. \quad (3.7)$$

Note that the brane pictures are reliable if the separations between the branes are sufficiently large and the string coupling is small. We thus set $g_s \ll 1$ and $y_1, y_2 - y_1 > l_s$, but consider the physics for generic values of Δx , similar to the study in [21]. In the regime for which $\Delta x > l_s$ perturbative string theory is reliable, and we can use it to study some aspects of the brane dynamics. On the other hand, in the regime where Δx is too small, the brane pictures are misleading, since perturbative string theory is not reliable. In particular, for $\Delta x \ll g_s l_s$ we should use gauge dynamics at low energies.

3.2 Brane Dynamics

Several phenomena in gauge theory have simple analogs in the brane construction. The tilted branes break supersymmetry. Furthermore, they can be displaced between the NS' and $N_f - N_{f0}$ D6 branes in the w direction. This corresponds to the pseudo-moduli Φ_{22}

in gauge theory.³ When Δx is sufficiently large, gravitational attraction of the tilted D4 branes to the NS brane fixes these moduli at $w = 0$. In gauge theory, an analogous effect is the stabilization of Φ_{22} at the origin by the one-loop effective potential. Remarkably, it was observed [10, 24] that in a large class of brane constructions gravitational attraction to the NS brane predicts phenomena which are realized in the low energy gauge theory due to one-loop quantum effects.

The other pseudo-moduli in Fig. 2(b) are the N_{f0} deformations of the massless flavor branes between the NS' and the N_{f0} D6 branes. These correspond to the expectation values $\langle Z_j^j \rangle$ in gauge theory. The location of the N_{f0} D4 branes in w is not fixed by an attraction to the NS brane, since the NS and these D4 branes are mutually BPS. Indeed, the one-loop effective potential in gauge theory does not fix the pseudo-moduli Z , as we have seen in the previous section.

We are thus led to consider subleading effects in the perturbative string theory regime (The analogous effect in gauge theory is the two-loop effective potential we studied.). There are several effects here which play an important role. Let us first focus on the NS' brane in Fig. 2(b) and further concentrate on the dynamics of the end-points of the fourbranes ending on it.⁴ These are codimension-two objects in the world-volume theory of a type IIA fivebrane. To understand their interactions we can consider the effective theory in three space-time dimensions. In this theory the end-points of D4 branes correspond to localized sources giving rise to an electric field and some scalar fields, as in [29].⁵

More specifically, a single D4 brane ending on an NS' brane at w_0 and going out in the direction y gives rise, for large $|w - w_0|$, to the following fields (in the normalization

³More precisely, to the expectation values $\langle (\Phi_{22})_i^i \rangle$; non-diagonal expectation values of Φ can be seen in the brane pictures if one separates the D6 branes in the y direction.

⁴We thank David Kutasov for pointing out the importance of the end-points dynamics, and for very helpful and interesting discussions.

⁵Recalling that the world-volume theory of a fivebrane in IIA string theory does not contain vector fields it may be confusing that it appears (sourced by the end-point) after dimensional reduction. The point is that the six-dimensional theory contains five scalars. Four are the usual Goldstone modes which encode the shape of the fivebrane in ten dimension and the fifth is a compact scalar which has to do with the M-theory circle. This compact scalar has a monodromy around the D4 end-point. Upon reducing to three dimensions we can use Poincarè duality and turn this vortex source into a usual local electric source for an Abelian three-dimensional gauge field. For a related analysis see [30]. We are grateful to Ofer Aharony for very helpful and interesting discussions.

of [17]):

$$y = g_s l_s \ln |w - w_0|, \quad A_0 = \frac{1}{l_s} \ln |w - w_0|. \quad (3.8)$$

A fourbrane going out at an angle θ (like the tilted D4 branes in Fig. 2(b)) from w'_0 has the following profile (as follows by rotational invariance in the xy plane):

$$y = g_s l_s \cos \theta \ln |w - w'_0|, \quad x = g_s l_s \sin \theta \ln |w - w'_0|, \quad A_0 = \frac{1}{l_s} \ln |w - w'_0|. \quad (3.9)$$

Since the world-volume theory of a single fivebrane is free we can use the superposition principle to construct a solution for two such D4 branes,

$$\begin{aligned} y &= g_s l_s (\ln |w - w_0| + \cos \theta \ln |w - w'_0|), & x &= g_s l_s \sin \theta \ln |w - w'_0|, \\ A_0 &= \frac{1}{l_s} (\ln |w - w_0| + \ln |w - w'_0|). \end{aligned} \quad (3.10)$$

It is straightforward to compute the binding energy of the system. Of course, scalars of like charges attract while identical electric charges repel. When $\theta = 0$ the system is BPS and the forces conspire to cancel. A non-zero relative angle does not affect the electrostatic force, as is evident in (3.10), but it decreases the attractive force from the exchange of y bosons. There is no overlap in the x direction so there is no binding force from exchanges of x . Hence, the end-points repel. The magnitude of the repelling force behaves, for large $|\Delta w|$, like

$$F(\Delta w) \simeq \frac{g_s^2 (1 - \cos \theta)}{l_s |\Delta w|}, \quad (3.11)$$

where $\Delta w = w_0 - w'_0$.

Evidently, there are other forces acting in the system. In general, separated non-parallel D branes in flat space always attract since gravity dominates the RR repulsion. So, far away from the NS' brane our N_{f0} D4 branes may feel some attraction. However, the dominant effect near the NS' brane is expected to be the Coulomb repulsion (3.11). To understand better the dynamics of this system one should solve the full non-linear DBI action (which should shed light on the short distance modifications of this Coulomb repulsion) as well as analyzing better the closed string interactions involved. Nevertheless, the considerations above strongly suggest that the end-points repel each other and the origin at $w = 0$ is destabilized.

To recapitulate, we presented an argument that the N_{f0} D4 branes in Fig. 2(b) are destabilized in the brane dynamics regime, nicely matching the field theory expectations. The analysis in the perturbative brane regime is straightforward and transparent compared

to the intricate two-loop computation needed in the gauge theory. However, the classical analysis above is not complete, but only presents some evidence for what appears to be the correct dynamics. It will be nice to perform more complete analysis of the various effects we described above and to obtain quantitative predictions for the fate of this system for generic separations of the brane.

4. Comments on General Distributions of Masses

There are some detailed implications of the results we obtained in the previous sections, but here we restrict ourselves to some qualitative features and postpone the complete phenomenological analysis to the future. Consider massive SQCD with N_f quarks in the free magnetic phase ordered as

$$0 < m_1 \leq m_2 \dots \leq m_{N_f}, \quad (4.1)$$

where we take the mass matrix to be diagonal with positive real eigenvalues m_i . We are interested in estimating how large should the hierarchy be, and among which masses, such that the model is destabilized.

By the Seiberg duality we arrive at the theory of subsection 2.2 with the superpotential

$$\mathcal{W} = hq\Phi\tilde{q} - h \sum_{i=1}^{N_f} \mu_i^2 \Phi_{ii}, \quad (4.2)$$

where $\mu_i^2 = m_i \Lambda$ and Λ is a strong coupling scale. If $\mu_1 = 0$ then Φ_{11} is not lifted at one-loop, as we have seen. Thus, the one-loop mass of Φ_{11} must be proportional to μ_1 . On the other hand, there is a non-vanishing two-loop contribution. We know that it must be proportional to a combination of μ_1, \dots, μ_{N_c} for the simple reason that if they all vanish the minimum is supersymmetric and the two-loop contribution vanishes. The most dominant two-loop contribution comes from μ_{N_c} .

We conclude that what is expected to affect the question of stability is primarily the ratio of μ_{N_c} and μ_1 . The suppressing factor is, naively, $\frac{4\pi}{h}$, the inverse loop expansion parameter. The correct suppression factor is supposedly even smaller due to the loop coefficients we calculated.⁶

In the perturbative brane dynamics regime a similar conclusion is made by very geometric and explicit means. The theory with general masses in the magnetic description has

⁶For non-zero μ_1 we expect the two-loop result to contain dependence on the renormalization scale which can render a precise estimate more complicated.

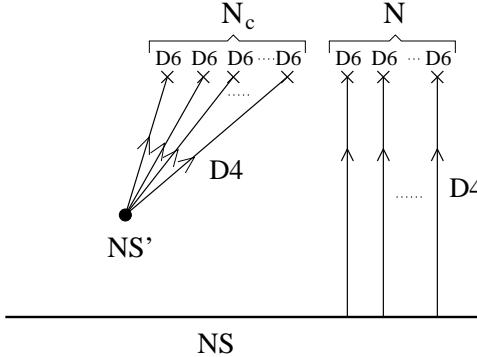


Figure 3: The embedding of ISS with general masses into string theory. The $N = N_f - N_c$ heavier flavors correspond to the vertical D4 branes, while the N_c lighter flavors correspond to the tilted D4 branes, one of whose ends lie on the NS' brane. As the mass of a light flavor is decreased, the D4 approaches a vertical line.

a brane embedding shown in Fig. 3. Now there are two competing forces: the attraction of the tilted branes to the NS and the repulsion among them. We shall call the tilted D4 brane corresponding to the i 'th flavor the μ_i brane, $i = 1, \dots, N_c$. The strength of the gravitational attraction of the μ_i brane to the NS is dictated by μ_i , as follows from eq. (3.7). Hence, the μ_1 brane experiences the smallest attraction to the NS. On the other hand, it is repelled from the other branes ending on the NS'. The largest repulsion is due to its interaction with the μ_{N_c} brane, since the angle between them is the largest. It follows from eqs. (3.5,3.7,3.11) that the strength of this repulsion is dictated by $h\mu_{N_c}$. Thus, qualitatively, we see the same behavior as in the gauge theory: the stability of the brane configuration is dictated by the ratio of $h\mu_{N_c}$ and μ_1 .

5. Summary

We have analyzed several aspects of $SU(N_c)$ SQCD with $N_f - N_{f0}$ massive flavors and N_{f0} massless flavors in the range $0 < N_{f0} < N_c < N_f < 3N_c/2$. Through a two-loop computation (which was made feasible using some simplifying observations and proper account of remnants of supersymmetric non-renormalization theorems), we found that the field theory is in a runaway phase with no meta-stable states near the origin of field space. We have also emphasized that our results may be important for model building inspired by ISS like scenarios, since one is often forced to make some hierarchy of masses to take care of the longevity – gaugino masses tension (or to fix some other phenomenological problems, e.g. Landau poles). As we have shown, the meta-stable minimum in massive

SQCD with hierarchical masses may be destabilized due to two-loop radiative corrections. It will be interesting to check what constraints are imposed by solving the above mentioned phenomenological problems without inducing instability.

A similar picture was obtained for the brane embedding of this model, though by much more elementary means. This provides an impressive test of the, yet mysterious, correspondence between the brane dynamics and gauge theory in SUSY breaking configurations. The brane dynamics can be applied to other systems, leading to new non-trivial “predictions” in gauge theory. For example, for a general mass distribution, as in section 4, there are various brane predictions regarding the two-loop results in gauge theory, and it will be nice to test them.

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A. Two-Loop Effective Potential

In this appendix we briefly review some of the results of the calculation of the two-loop effective potential [26], which are used in this work. As we show in subsection 2.2, the effects of gauging are irrelevant for our calculations in this paper. Hence, for simplicity we shall review here interacting theories of scalars and fermions.

Consider a model with a set of *real* scalars R_i and Weyl fermions ψ_I . The masses of these are given by

$$\mathcal{L}_{mass} = -\frac{1}{2}(m^2)^{ij}R_iR_j - \frac{1}{2}M^{IJ}\psi_I\psi_J + \text{c.c.} \quad (\text{A.1})$$

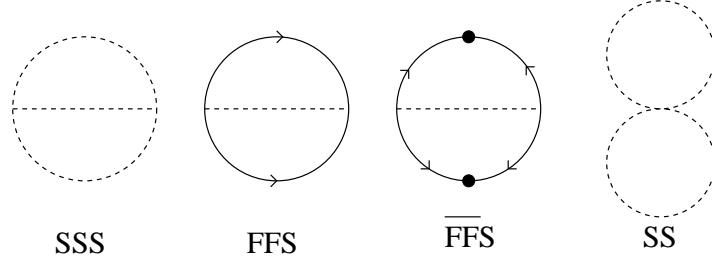


Figure 4: Four possible diagrams which contribute to the two-loop effective potential of a theory with interacting scalars (represented by dashed lines) and fermions (solid lines).

We consider a basis where the mass-squared matrices m_{ij}^2 and $M_{IJ}^2 \equiv M_{IK}^\dagger M_{KJ}$ are already diagonal, with eigenvalues m_i^2 and m_J^2 , respectively. Note that the (symmetric) fermionic matrix M_{IJ} is not necessarily diagonal.

The only possible renormalizable interactions in this theory are cubic and quartic interactions for the scalars and Yukawa interactions of two fermions and a scalar. Following the conventions of [26] we parameterize them as follows:

$$\mathcal{L}_{int} = -\frac{1}{6}\lambda^{ijk}R_iR_jR_k - \frac{1}{24}\lambda'^{ijkl}R_iR_jR_kR_l - \left(\frac{1}{2}Y^{IJK}\psi_I\psi_JR_k + \text{c.c.}\right). \quad (\text{A.2})$$

Note that the couplings λ and λ' are real and symmetric under the interchange of each pair of indices. The Yukawa couplings Y_{IJK} are symmetric under interchanges of spinor flavor indices I and J .

In the perturbative regime one can expand the effective potential as

$$V = V^{(0)} + \frac{1}{16\pi^2}V^{(1)} + \frac{1}{(16\pi^2)^2}V^{(2)} + \dots. \quad (\text{A.3})$$

Generically, the two-loop potential $V^{(2)}$ depends on the renormalization scale, Q . The four possible diagrams which can contribute to $V^{(2)}$ are depicted schematically in Fig. 4. We will further refer to these diagrams as SSS, FFS, \overline{FFS} and SS respectively. Note that the diagram \overline{FFS} appears since the masses of fermions are not necessarily diagonal.

The contribution of each of these diagrams is parameterized by the following functions:

$$V_{\text{SSS}}^{(2)} = \frac{1}{12}(\lambda^{ijk})^2 f_{\text{SSS}}(m_i^2, m_j^2, m_k^2) \quad (\text{A.4})$$

$$V_{\text{SS}}^{(2)} = \frac{1}{8}\lambda^{iijj} f_{\text{SS}}(m_i^2, m_j^2) \quad (\text{A.5})$$

$$V_{\text{FFS}}^{(2)} = \frac{1}{2}|Y^{IJK}|^2 f_{\text{FFS}}(m_I^2, m_J^2, m_k^2) \quad (\text{A.6})$$

$$V_{\overline{\text{FFS}}}^{(2)} = \frac{1}{4}Y^{IJK}Y^{I'J'K}M_{II'}^*M_{J'J}^*f_{\overline{\text{FFS}}}(m_I^2, m_J^2, m_k^2) + \text{c.c.} \quad (\text{A.7})$$

The functions f can be expressed in terms of three functions $I(x, y, z)$, $J(x, y)$ and $J(x)$ which are defined as

$$J(x) = x \left(\ln \frac{x}{Q^2} - 1 \right) \quad (\text{A.8})$$

$$J(x, y) = J(x)J(y) \quad (\text{A.9})$$

$$\begin{aligned} I(x, y, z) = & \frac{1}{2}(x - y - z) \ln \frac{y}{Q^2} \ln \frac{z}{Q^2} + \frac{1}{2}(y - x - z) \ln \frac{x}{Q^2} \ln \frac{z}{Q^2} + \\ & \frac{1}{2}(z - x - y) \ln \frac{x}{Q^2} \ln \frac{y}{Q^2} + 2x \ln \frac{x}{Q^2} + 2y \ln \frac{y}{Q^2} + \\ & 2z \ln \frac{z}{Q^2} - \frac{5}{2}(x + y + z) - \frac{1}{2}\xi(x, y, z), \end{aligned} \quad (\text{A.10})$$

where ξ is defined by

$$\begin{aligned} \xi(x, y, z) = & R \left(2 \ln \frac{z + x - y - R}{2z} \ln \frac{z + y - x - R}{2z} - \ln \frac{x}{z} \ln \frac{y}{z} - \right. \\ & \left. 2 \text{Li}_2 \frac{z + x - y - R}{2z} - 2 \text{Li}_2 \frac{z + y - x - R}{2z} + \frac{\pi^2}{3} \right), \end{aligned} \quad (\text{A.11})$$

with

$$R = \sqrt{x^2 + y^2 + z^2 - 2xy - 2xz - 2yz}. \quad (\text{A.12})$$

In terms of I and J , the functions f are given by

$$f_{\text{SSS}}(x, y, z) = -I(x, y, z) \quad (\text{A.13})$$

$$f_{\text{SS}}(x, y) = J(x, y) \quad (\text{A.14})$$

$$f_{\text{FFS}}(x, y, z) = J(x, y) - J(x, z) - J(y, z) + (x + y - z)I(x, y, z) \quad (\text{A.15})$$

$$f_{\overline{\text{FFS}}}(x, y, z) = 2I(x, y, z). \quad (\text{A.16})$$

In our specific model we need only I functions with at least one argument vanishing, so we give them explicitly

$$\begin{aligned} I(0, x, y) = & (x - y) \left(\text{Li}_2(y/x) - \ln(x/y) \ln \frac{x - y}{Q^2} + \frac{1}{2}(\ln \frac{x}{Q^2})^2 - \frac{\pi^2}{6} \right) \\ & - \frac{5}{2}(x + y) + 2x \ln \frac{x}{Q^2} + 2y \ln \frac{y}{Q^2} - x \ln \frac{x}{Q^2} \ln \frac{y}{Q^2}. \end{aligned}$$

In the case that two arguments vanish it simplifies further

$$I(0, 0, x) = -\frac{1}{2}x(\ln \frac{x}{Q^2})^2 + 2x \ln \frac{x}{Q^2} - \frac{5}{2}x - \frac{\pi^2}{6}x.$$

However, the expression for $I(0, x, y)$ is still not very convenient since it contains terms which have no Taylor expansion around the point $x = y$, which appears commonly in our

expressions. This is a spurious singularity which cancels once all the terms are summed. To remove it once and for all we use Euler's identity for Dilogarithms

$$\text{Li}_2(x) + \text{Li}_2(1-x) = -\ln x \ln(1-x) + \frac{\pi^2}{6},$$

which gives

$$\begin{aligned} I(0, x, y) &= (x-y) \left(-\text{Li}_2(1-y/x) - \ln(x/y) \ln \frac{x}{Q^2} + \frac{1}{2} (\ln \frac{x}{Q^2})^2 \right) \\ &\quad - \frac{5}{2}(x+y) + 2x \ln \frac{x}{Q^2} + 2y \ln \frac{y}{Q^2} - x \ln \frac{x}{Q^2} \ln \frac{y}{Q^2}. \end{aligned}$$

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